

Current status of ε_K in lattice QCD

Weonjong Lee

Director, Lattice Gauge Theory Research Center, CTP, and FPRD,
Department of Physics and Astronomy,
Seoul National University, Seoul 08826, South Korea

E-mail: wlee@snu.ac.kr

Abstract. We present the current status of ε_K evaluated directly from the standard model using lattice QCD inputs. The lattice QCD inputs include \hat{B}_K , ξ_0 , ξ_2 , $|V_{us}|$, $m_c(m_c)$, and $|V_{cb}|$. Recently, FLAG has updated \hat{B}_K , exclusive $|V_{cb}|$ has been updated with new lattice data in the $\bar{B} \rightarrow D\ell\bar{\nu}$ decay mode, and RBC-UKQCD has updated ξ_0 and ξ_2 . We find that the standard model evaluation of ε_K with exclusive $|V_{cb}|$ (lattice QCD inputs) is 3.2σ lower than the experimental value, while that with inclusive $|V_{cb}|$ (heavy quark expansion) shows no tension.

1. Introduction

Since 2012, we have been monitoring ε_K , the indirect CP violation parameter in neutral kaons using lattice QCD inputs. The parameter ε_K is, in particular, very attractive to the particle physics community, since it is very precisely measured in experiment, and it provides a direct probe of CP violation in the standard model and in physics models beyond the standard model (BSM). In this paper, we present results of ε_K evaluated directly from the standard model with lattice QCD inputs and compare them with the experimental value. This paper is an update of our previous paper [1, 2].

2. Input parameters

In the standard model, the indirect CP violation parameter of the neutral kaon system ε_K can be expressed as follows,

$$\begin{aligned}\varepsilon_K &\equiv \frac{\mathcal{A}(K_L \rightarrow \pi\pi(I=0))}{\mathcal{A}(K_S \rightarrow \pi\pi(I=0))} \\ &= e^{i\theta} \sqrt{2} \sin \theta \left(C_\varepsilon X_{\text{SD}} \hat{B}_K + \frac{\xi_0}{\sqrt{2}} + \xi_{\text{LD}} \right) + \mathcal{O}(\omega\varepsilon') + \mathcal{O}(\xi_0\Gamma_2/\Gamma_1).\end{aligned}\quad (1)$$

Here, the short distance contribution proportional to \hat{B}_K occupies about 105% of ε_K , the long distance effect from the absorptive part ξ_0 gives about -5% correction, and the long distance effect from the dispersive part ξ_{LD} gives about $\pm 1.6\%$ correction. The details on C_ε , X_{SD} , ξ_0 , and ξ_{LD} are described in Ref. [1]. In order to determine ε_K directly from the standard model, we need 18 input parameters, and 6 of them can, in principle, be obtained from lattice QCD: V_{us} , V_{cb} , \hat{B}_K , ξ_0 , ξ_{LD} , and $m_c(m_c)$. Here, we address recent progress on determining those input parameters.

Table 1: Results for $|V_{cb}|$

Decay mode	$ V_{cb} $	Ref.
$\bar{B} \rightarrow D^* \ell \bar{\nu}$	39.04(49)(53)(19)	[3]
$\bar{B} \rightarrow D \ell \bar{\nu}$	40.7(10)(2)	[4]
ex-combined	39.62(60)	this paper
$\bar{B} \rightarrow X_c \ell \bar{\nu}$	42.00(64)	[5]

Table 2: Results for $|V_{ub}|$

Decay mode	$ V_{ub} $	Ref.
$\bar{B} \rightarrow \pi \ell \bar{\nu}$	3.72(16)	[6]
$\bar{B} \rightarrow \pi \ell \bar{\nu}$	3.61(32)	[7]
ex-combined	3.70(14)	this paper
$\bar{B} \rightarrow X_u \ell \bar{\nu}$	4.45(16)(22)	[8]

2.1. $|V_{cb}|$

Recent results for $|V_{cb}|$ and $|V_{ub}|$ are presented in Tables 1 and 2. Recently, DeTar has collected the results of $\bar{B} \rightarrow D \ell \bar{\nu}$ decay at non-zero recoil from both lattice QCD [9, 10] and experiments of Babar [11] and Belle [12], and has made a combined fit of all the data simultaneously to determine $|V_{cb}|$ [4]. We have obtained the “ex-combined” result in Table 1 by taking a weighted average of the V_{cb} results from the $\bar{B} \rightarrow D^* \ell \bar{\nu}$ and $\bar{B} \rightarrow D \ell \bar{\nu}$ decay channels. Similarly, we have obtained the “ex-combined” result in Table 2 by taking a weighted average of the two V_{ub} results from $\bar{B} \rightarrow \pi \ell \bar{\nu}$ decay. In Fig. 1, we show all the results in various colors.¹ The inclusive results are about 3σ away from those from exclusive B meson decays respectively as well as from the LHCb results for $|V_{ub}/V_{cb}|$, which corresponds to the magenta band in Fig. 1.

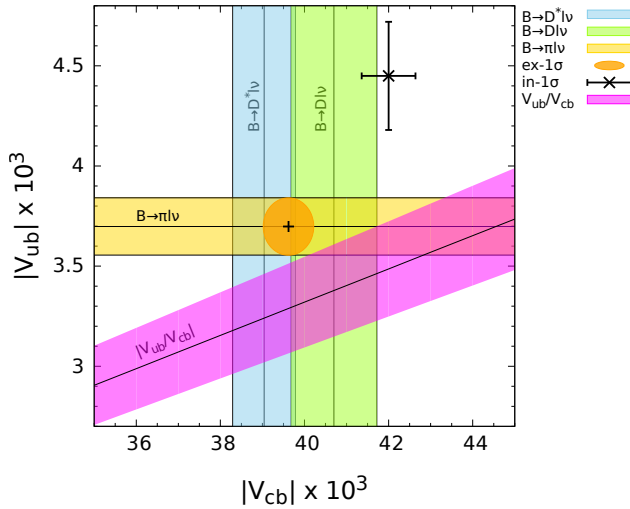


Figure 1: $|V_{cb}|$ versus $|V_{ub}|$. The sky-blue band represents $|V_{cb}|$ determined from the $\bar{B} \rightarrow D^* \ell \bar{\nu}$ decay mode. The yellow-green band represents $|V_{cb}|$ determined from the $\bar{B} \rightarrow D \ell \bar{\nu}$ decay mode. The yellow band represents $|V_{ub}|$ determined from the $\bar{B} \rightarrow \pi \ell \bar{\nu}$ decay mode. The magenta band represents $|V_{ub}/V_{cb}|$ determined from the LHCb data of the $\Lambda_b \rightarrow \Lambda_c \ell \bar{\nu}$ and $\Lambda_b \rightarrow p \ell \bar{\nu}$ decay modes. The orange circle represents the combined results for exclusive $|V_{cb}|$ and $|V_{ub}|$ from the B meson decays within 1.0σ . The black cross (\times) represents the inclusive $|V_{cb}|$ and $|V_{ub}|$ from the heavy quark expansion. The details are given in Tables 1 and 2.

2.2. ξ_0 and ξ_{LD}

There are two independent methods to determine ξ_0 in lattice QCD: the indirect and direct methods. In the indirect method, we determine ξ_0 from the experimental values of $\text{Re}(\varepsilon'/\varepsilon)$, ε_K , ω and the lattice QCD input ξ_2 . They are related to one another as follows,

$$\xi_0 = \frac{\text{Im}A_0}{\text{Re}A_0}, \quad \xi_2 = \frac{\text{Im}A_2}{\text{Re}A_2}, \quad \text{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \frac{\omega}{\sqrt{2}|\varepsilon_K|}(\xi_2 - \xi_0). \quad (2)$$

¹ The plot is based on that by Andreas Kronfeld in Ref. [4].

Table 3: ξ_0 and ξ_{LD} .

Input	Method	Value	Ref.
ξ_0	indirect	$-1.63(19) \times 10^{-4}$	[13]
ξ_0	direct	$-0.57(49) \times 10^{-4}$	[14]
ξ_{LD}	—	$(0 \pm 1.6) \%$	[15]

Table 4: δ_0

Collaboration	δ_0	Ref.
RBC-UK-2016	$23.8(49)(12)^\circ$	[14]
KPY-2011	$39.1(6)^\circ$	[16]
CGL-2001	$39.2(15)^\circ$	[17, 18]

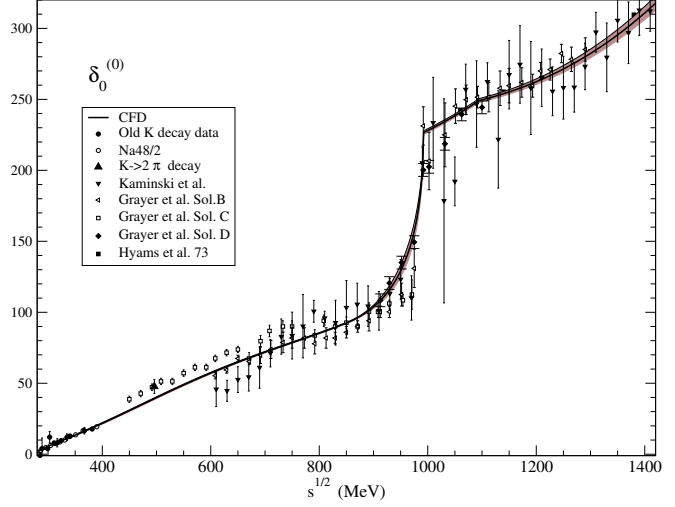


Figure 2: δ_0 from KPY-2011.

Recently, RBC-UKQCD reported updated results for ξ_2 [13]. The results for ξ_0 from the indirect method are given in Table 3.

Recently, RBC-UKQCD has reported new lattice QCD results for $\text{Im}A_0$ [14]. Combining their results with the experimental value of $\text{Re}A_0$, we can determine ξ_0 directly from the lattice input $\text{Im}A_0$. This is the direct method. In Ref. [14], RBC-UKQCD has also reported the S-wave $\pi - \pi$ scattering phase shift with isospin $I = 0$: $\delta_0 = 23.8(49)(12)$. This value has 3.0σ tension with the conventional determination of δ_0 in Refs. [16] (KPYP-2011) and [17, 18] (CGL-2001). They used a singly subtracted Roy-like equation (KPYP-2011) or a doubly subtracted Roy equation (CGL-2001) to do the interpolation around $\sqrt{s} = m_K$ (kaon mass). The values for δ_0 are summarized in Table 4. In Fig. 2, we show the fitting results of KPY-2011. Their fits to the experimental data work well from the $\pi - \pi$ threshold to $\sqrt{s} = 800$ MeV, and are highly consistent with CGL-2001 in the interpolating region around $\sqrt{s} = m_K \approx 500$ MeV.

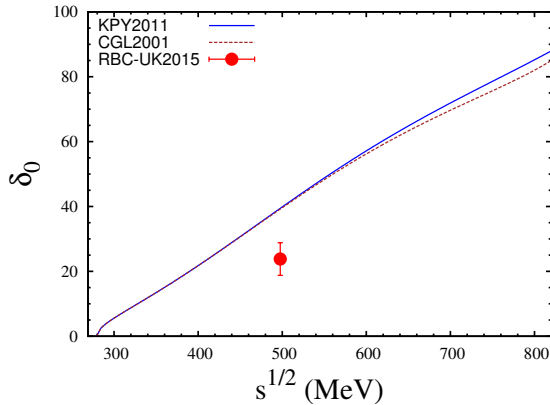


Figure 3: Comparison of δ_0 .

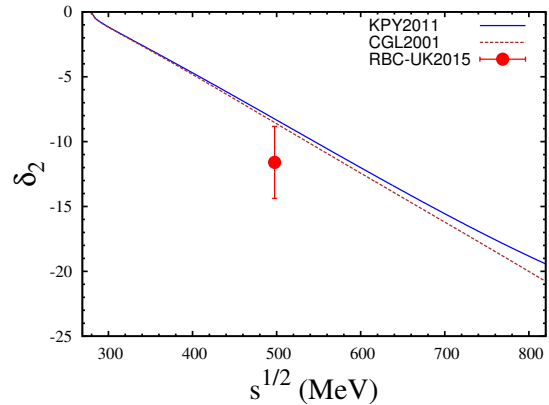


Figure 4: Comparison of δ_2 .

In Fig. 3, we present the results of RBC-UKQCD together with the fitting results of KPY-2011 and CGL-2001. There is essentially no difference between KPY-2011 and CGL-2001 in the

region near $\sqrt{s} = m_K \approx 500 \text{ MeV}$. Here, we observe the 3.0σ gap between RBC-UKQCD and KPY-2011. In contrast, for δ_2 (S-wave, $I=2$), there is no difference between RBC-UKQCD and KPY-2011 within statistical uncertainty, as one can see in Fig. 4.

Therefore, we conclude that the results of the indirect method are more reliable than those of the direct method for ξ_0 , since the direct calculation of $\text{Im}A_0$ by RBC-UKQCD might have unresolved issues. Hence, we use the indirect method to determine ξ_0 in this paper.

Regarding ξ_{LD} , the long distance effect in the dispersive part, the theoretical master formula in the continuum is given in Ref. [1]. A theoretical framework for calculating it on the lattice is well established in Ref. [15]. There has been an on-going attempt to calculate it on the lattice [19]. However, this attempt [20], at present, is not mature and belongs to the category of exploratory study rather than to that of precision measurement. Hence, we use a rough estimate of ξ_{LD} given in Ref. [19] in this paper. It is summarized in Table 3.

2.3. \hat{B}_K

In Table 5, we present results for \hat{B}_K calculated using lattice QCD tools with $N_f = 2+1$ flavors. Here, FLAG-2016 represents the global average of the results of BMW-2011 [21], Laiho-2011 [22], RBC-UK-2016 [23], and SWME-2016 [24], which is summarized in Ref. [25]. SWME-2014 represents the \hat{B}_K result reported in Ref. [26]. RBC-UK-2016 represents that reported in Ref. [23].

The results of SWME-2016 are obtained using fitting based on staggered chiral perturbation theory (SChPT) in the infinite volume limit, and those of SWME-2014 are obtained using fitting based on SChPT with finite volume corrections incorporated at the NLO level. Here we use the FLAG-2016 result for \hat{B}_K .

Table 5: \hat{B}_K .

Collaboration	Value	Ref.
FLAG-2016	0.7625(97)	[25]
SWME-2014	0.7379(47)(365)	[26]
RBC-UK-2016	0.7499(24)(150)	[23]

Table 6: Wolfenstein parameters.

	CKMfitter	UTfit	AOF
λ	0.22548(68)/[27]	0.22497(69)/[28]	0.2253(8)/[29]
$\bar{\rho}$	0.145(13)/[27]	0.153(13)/[28]	0.139(29)/[30]
$\bar{\eta}$	0.343(12)/[27]	0.343(11)/[28]	0.337(16)/[30]

2.4. Other input parameters

For the Wolfenstein parameters λ , $\bar{\rho}$, and $\bar{\eta}$, both CKMfitter and UTfit updated their results in Refs. [27, 28], but the angle-only-fit has not been updated since 2015. The global unitarity triangle (UT) fits of both CKMfitter and UTfit use ε_K and $|V_{cb}|$ as input parameters to determine the apex $\bar{\rho}$ and $\bar{\eta}$. Hence, using them to evaluate ε_K introduces unwanted correlations through ε_K and $|V_{cb}|$. In contrast, the angle-only-fit (AOF) results are independent of ε_K and $|V_{cb}|$. Hence, we use the AOF results in this paper.

For the QCD corrections η_{cc} , η_{ct} , and η_{tt} , we use the same values as in Ref. [1], which are given in Table 7. In particular, we use the SWME value of η_{cc} reported in Ref. [1] instead of that in Ref. [31]. This issue is well explained in Ref. [1]. One of the reasons is that the size of the NNLO correction is already a conservative estimate for the truncation error of the NNNLO level in perturbation theory. Another reason is that the SWME result is highly consistent with that of Ref. [32].

In Table 8, we summarize other input parameters. They are the same as those in Ref. [1] except for the charm quark mass $m_c(m_c)$. For the charm quark mass, we use the HPQCD result reported in Ref. [35].

Table 8: Other input parameters.

Table 7: QCD corrections.

Input	Value	Ref.
G_F	$1.1663787(6) \times 10^{-5} \text{ GeV}^{-2}$	[29]
M_W	80.385(15) GeV	[29]
$m_c(m_c)$	1.2733(76) GeV	[35]
$m_t(m_t)$	163.3(2.7) GeV	[36]
θ	$43.52(5)^\circ$	[29]
m_{K^0}	497.614(24) MeV	[29]
ΔM_K	$3.484(6) \times 10^{-12} \text{ MeV}$	[29]
F_K	156.2(7) MeV	[29]

3. Current status of ε_K

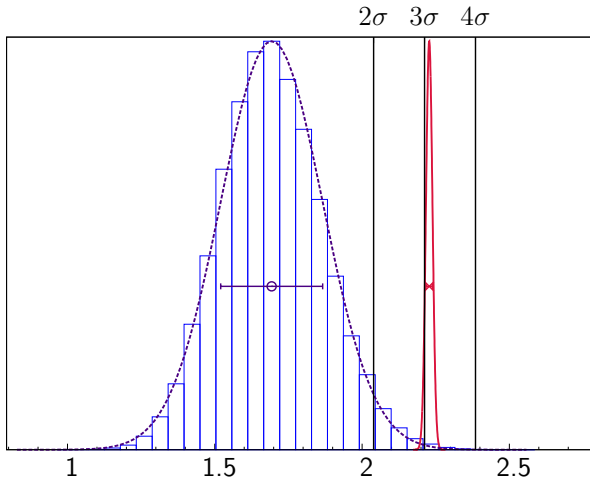
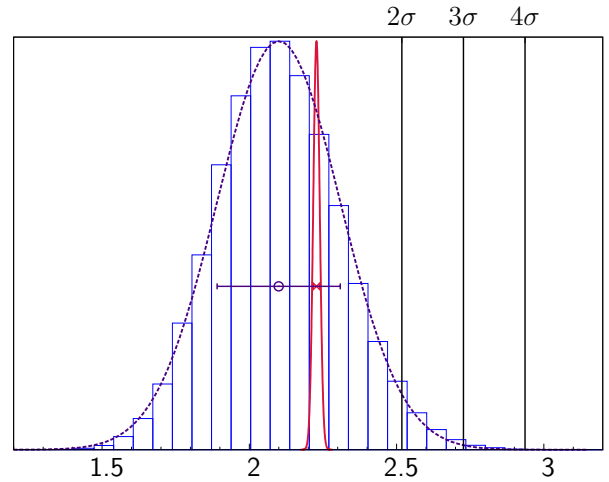
In Fig. 5, we show the results for ε_K evaluated directly from the standard model with the lattice QCD inputs described in the previous section. Here, the blue curve represents the theoretical evaluation of ε_K with the FLAG-2016 \hat{B}_K , AOF for the Wolfenstein parameters, and exclusive $|V_{cb}|$ that corresponds to ex-combined in Table 1. The red curve represents the experimental value of ε_K . In Fig. 6, the blue curve represents the same as that in Fig. 5 except for using the inclusive $|V_{cb}|$ in Table 1. Our preliminary results are, in units of 1.0×10^{-3} ,

$$|\varepsilon_K| = 1.69 \pm 0.17 \quad \text{for exclusive } V_{cb} \text{ (lattice QCD)} \quad (3)$$

$$|\varepsilon_K| = 2.10 \pm 0.21 \quad \text{for inclusive } V_{cb} \text{ (QCD sum rules)} \quad (4)$$

$$|\varepsilon_K| = 2.228 \pm 0.011 \quad \text{(experimental value)} \quad (5)$$

We find that there is 3.2σ tension in the exclusive V_{cb} channel (lattice QCD), and no tension in the inclusive V_{cb} channel (heavy quark expansion; QCD sum rules).

Figure 5: ε_K with exclusive V_{cb} .Figure 6: ε_K with inclusive V_{cb} .

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